Snowmelt and Frozen Soil
[Based on class notes by Erin Brooks and the text by Dingman (2002)]

I. Snow Hydrology

Terms

**Precipitation**: the depth of rainfall plus the water equivalent of snow, sleet, and hail falling during a given storm or measurement period

**Snowfall**: the incremental depth of snow and other forms of solid precipitation accumulating on the surface during a given storm or measurement period

**Snowcover/snowpack**: the entire accumulated snow on the ground at a time of measurement

**Snowmelt**: the amount of liquid water produced by melting that leaves the snowpack during a given time period

Occurrence

Globally, all the land above 40° N latitude has a seasonal snow cover of significant duration, and snowfall contributes more than rainfall to runoff in many areas. In the Colorado Rockies about 90% of the yearly water supply is derived from snowfall. In the Bitterroot Mountains, approximately 50% and 70% of total yearly precipitation falls as snow at an elevation of 5,000- and 7,000 ft, respectively. In the Moscow–Pullman area, approximately 20% of total precipitation falls as snow (see Table 1).

Table 1. Average annual precipitation for Moscow, Pullman and vicinity.

<table>
<thead>
<tr>
<th></th>
<th>Pullman</th>
<th>Moscow</th>
<th>Spokane</th>
<th>Lewiston</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. Ann. Precip. (in.)</td>
<td>21.5</td>
<td>23.5</td>
<td>16.1</td>
<td>12.8</td>
</tr>
<tr>
<td>Avg. Ann. Snowfall (in.)</td>
<td>28.5</td>
<td>49.5</td>
<td>41.8</td>
<td>15.8</td>
</tr>
<tr>
<td>Elevation (ft)</td>
<td>2,540</td>
<td>2,660</td>
<td>2,360</td>
<td>1,440</td>
</tr>
</tbody>
</table>

Mt. Baker, 56 mi. east of Bellingham, WA, with an elevation of 4,200 ft, recorded a world record total snowfall of 1,140 inches (95 ft) for the 1998–99 season. Its average annual snowfall is 645 inches (54 ft), the highest average value in North America.

Importance of the Study of Snow Hydrology

- Flood forecasting
- Reservoir management
- Water supply and irrigation design
- Hydrologic modeling of watershed processes (surface runoff, streamflow, sediment transport, nutrient transport, depth of frozen soil)
- General design (highways, culverts)
- Safety and recreation (avalanche warnings, ski and road conditions forecast)
General Composition and Characteristics

Snow is a granular porous medium consisting of ice and pore spaces. When it is cold (temperature below 0 °C), the pore spaces contain only air (including water vapor). At the melting point, the pore spaces can contain liquid water and air, and snow becomes a three-phase system. In most practical applications, the actual depth of the snowpack is not as important as the depth of the snow water equivalent ($SWE$) of the snowpack

$$SWE = \frac{\rho_s h_s}{\rho_w}$$

(1)

where

- $h_s =$ height of snowpack
- $\rho_s =$ density of snow
- $\rho_w =$ density of water

Note that snowpack density may change with time and snow density within a snowpack changes with depth. In the later section on snow properties, we will revisit the term $SWE$.

The density of snow is typically measured with a weighing snow gage. The density of snowpack changes due to rainfall, settling, metamorphism due to temperature and vapor gradients. Snowpack density will vary between 0.15 g cm$^{-3}$ for new pack and 0.45 g cm$^{-3}$ for a melting pack.

The density of new fallen snow is determined by the configuration of the snowflakes, which is largely a function of air temperature, the degree of super-saturation in the precipitating cloud, and the wind speed at the surface of deposition. Because of the difficulty of measuring the density of new snow, an average density of 0.10 g cm$^{-3}$ is often assumed. Since the density of water is 1 g cm$^{-3}$, a rule of thumb is that, for new fallen snow, the $SWE$ depth will be roughly 10% of the actually measured snow depth. In other words, 10 inches of snow is equivalent to 1 inch of water.

Snowpack Metamorphism

1. **Gravitational settling (rate)**
   - increases with overlying snow weight and the temperature of the layer
   - decreases with the layer’s density

2. **Destructive metamorphism**
   - vapor pressures higher over more curved surface, causing the points and projections of snowflakes to evaporate and deposit on nearby less convex surfaces, thus forming larger, more spherical snow grains
   - The process occurs most rapidly in recently fallen snowflakes and ceases when the snow density becomes high (~ 0.25 g cm$^{-3}$)

3. **Constructive metamorphism**
   - most important pre-melt densification process
   - Over short distances: the process occurring by sintering (water molecules deposit in concavities where snow grains touch and gradually build a “neck” between adjacent grains
   - Over long distances: the process occurring via vapor transfer due to temperature gradients
   - “Depth hoar”: forming when very cold air overlying a shallow snowpack produces an upward-decreasing temperature and vapor pressure gradient, and snow near the base evaporates
resulting in a basal layer of planar crystals with low density and strength

4. Melt metamorphism
- densification by liquid water (from surface melting or as rainfall) and formation of solid ice
- disappearance of smaller snow grains and growth of larger grains

All processes except the temporary “depth hoar” increase the snow density through the snow-accumulation season.

Snowmelt Processes

1. **Accumulation period**: the period of general increase of snowpack water equivalent prior to the melt period

2. **Melt period**: the period when the net energy input to a snowpack becomes more or less continually positive, including three phases:
   
   (1) *Warming*: during which the average snowpack temperature increases until the snowpack is isothermal at 0 °C
   
   (2) *Ripening*: during which melting occurs but the melt water is retained in the snowpack that is isothermal at 0 °C until it cannot retain any more liquid water and is said to be **ripe**
   
   (3) *Output*: during which further energy inputs produce water output

**Field Measurement**

Snowfall measurement
- Standard method: ruler
- Universal gage: recording weight increase
- Radar: distinguish rain and snow; delineate the areal extent

Snowcover measurement
- Snow stakes or rulers
- Snow survey via a snow course
- Snow pillows: made of rubber containing liquid with low freezing point; measuring pressure
- Radioactive gages: recording the attenuation of gamma rays or neutrons by water
- Microwave: measuring the areal extent

Snowmelt measurement
- Lysimeters: collect the water draining from the overlying snow and then measure the flow

**Snow Properties**

Let us consider a representative portion of a snowpack of height \( h_s \) and area \( A \). Using the symbols \( M \) to designate mass, \( V \) for volume, \( h \) for height, and \( \rho \) for mass density; and the subscripts \( s \) for snow, \( i \) for ice, \( w \) for liquid water, \( m \) for water substance, \( a \) for air; we can derive the following relationships. The snow volume is

\[
V_s = V_i + V_w + V_a = h_s \cdot A
\]
Fig. 1. Dimension of a representative portion of a snowpack. \( A \) is the area of the upper surface; \( h_s \) is the snow depth. [Adapted from Dingman (2002, Fig. 5-1).]

The porosity, \( \phi \), is defined as the ratio of pore volume to total volume

\[
\phi = \frac{V_A + V_w}{V_s}
\]  

(3)

Therefore,

\[
V_s = (1 - \phi) \cdot V_s
\]  

(4)

The liquid water content, \( \theta \), is defined as the ratio of the volume of liquid water in the snowpack to the total volume of snow:

\[
\theta = \frac{V_w}{V_s}
\]  

(5)

Snow density, \( \rho_s \), is defined as the mass per unit volume of snow, so

\[
\rho_s = \frac{M_s}{V_s} = \frac{\rho_i \cdot V_i + \rho_w \cdot V_w}{V_s}
\]  

(6)

Combining Eq 3–6 allows us to relate density, liquid water content, and porosity as

\[
\rho_s = (1 - \phi) \cdot \rho_i + \theta \cdot \rho_w \quad \text{or} \quad \frac{\rho_s}{\rho_w} = \theta + (1 - \phi) \frac{\rho_i}{\rho_w}
\]  

(7)

where \( \rho_i = 917 \text{ kg m}^{-3} \) and \( \rho_w = 1,000 \text{ kg m}^{-3} \).

As we mentioned earlier, for the hydrologist, the most important property of a snowpack is the SWE,
often expressed as the depth of water that would result from the complete melting of the snow in place, \( h_m \). Thus,

\[
h_m = \frac{V_m}{A} \tag{8}
\]

where \( V_m \) is the volume of water resulting from the complete melting of the snow with the same surface area of \( A \), and it is related to the volume of water and ice following

\[
V_m = V_w + \frac{\rho_i}{\rho_w} \cdot V_i
\tag{9}
\]

(Why?)

Substituting Eq 4 and 5 into Eq 9 yields

\[
V_m = \theta \cdot V_s + (1 - \phi) \cdot V_s \cdot \frac{\rho_i}{\rho_w}
\tag{10}
\]

Using Eq 2 and 8 to rewrite Eq 10 and dividing both sides by \( A \) gives

\[
h_m = \theta \cdot h_s + (1 - \phi) \cdot h_s \cdot \frac{\rho_i}{\rho_w} = \left[ \theta + (1 - \phi) \cdot \frac{\rho_i}{\rho_w} \right] \cdot h_s
\tag{11}
\]

Finally, we see from Eq 7 that Eq 11 can be written as

\[
h_m = \frac{\rho_s}{\rho_w} \cdot h_s
\tag{12}
\]

(Why?) Note that this is the same equation as Eq 1. Do you have another way deriving it?

**Snowmelt Period Energy Needs**

1. Warming phase

The cold content, \( Q_c \) (of dimension \([\text{E L}^{-2}]\)), of a snowpack is the amount of energy required to raise its average temperature to the melting point, and

\[
Q_c = -c_i \cdot \rho_w \cdot h_m \cdot (T_s - T_m)
\tag{13}
\]

where \( c_i \) is the heat capacity of ice (2,102 J kg\(^{-1}\) K\(^{-1}\)), \( T_s \) is the average temperature of the snowpack, \( T_m \) is the melting-point temperature (273.15 K), and the other symbols are as previously defined. The cold content can be computed at any time prior to the ripening phase, and the net energy input required to complete phase 1, \( Q_{m1} \), equals the cold content at the beginning of the melt period.

2. Ripening phase

The liquid water retaining capacity, \( h_{wret} \), of a snowpack, is given as
and, the net energy input required to complete the ripening phase, $Q_{m2}$, can be computed as

$$ Q_{m2} = \theta_{mf} \cdot h_s \cdot \rho_v \cdot \lambda_f $$

where $\lambda_f$ is the latent heat of fusion (0.334 MJ kg$^{-1}$).

(Define $\theta_{mf}$; understand Eq 14 by visualizing a snowpack)

3. Output phase

The net energy input required to complete the output phase, $Q_{m3}$, is the amount of energy needed to melt the snow remaining at the end of the ripening phase:

$$ Q_{m3} = (h_m - h_{wet}) \cdot \rho_v \cdot \lambda_f $$

The total energy need is the sum of the energy inputs for each melt phase.

$$ Q = Q_{m1} + Q_{m2} + Q_{m3} $$

Snowmelt Modeling over a Watershed

To accurately model the snow distribution and snowmelt over a large watershed with extensive differences in elevation and topography, information on spatially distributed air temperature and precipitation and other physical settings (e.g., soil type, geology, land use, ground cover) should be used.

Ideally, for a large watershed, air temperature and precipitation measurements will be available at more than one location and cover both low and high points. It is common to distribute both temperature and precipitation linearly by elevation. If only one weather station is present, it is common to assume the temperature decreases with elevation according to the adiabatic lapse rate of $-3.5$ °F per 1,000 ft.

One technique to simulate the snowmelt over a watershed with significant differences in elevation is to use a “lumped” approach. In this approach, a hypsometric curve (elevation vs. percent area) of the watershed is generated. The watershed is then broken into bands representing a fixed range of elevations. Snow accumulation and melt are calculated for each band and the combined response from each band is the total response of the watershed.

A major challenge in snowmelt modeling is a proper distribution of the snowfall across a landscape. The drifting snow can be seen across the Palouse region especially due to the rolling topography. Much of the early research in snow drifting has focused on the design of snow fences to minimize drifting across major highways (Tabler, 1975). Efforts have also been devoted to studying snow distribution in forested landscapes. Small forest openings (diameter < 20 times the height of surrounding trees) tend to have greater accumulation than the surrounding forest. While large forest openings tend to have less snow accumulation caused by wind with higher speeds. In recent years, researchers have been evaluating the impact of conservation tillage on trapping snow and maintaining high soil water in the seed zone.

GIS techniques are increasingly used in snow hydrologic modeling in order to more precisely simulate the spatial variation of snow accumulation and melt as affected by the numerous factors, such as vegetation
and surface cover conditions, across a large domain.

**Important References**

  - Historic and current SNOTEL and snow-course data (NWCC provides real-time snow and climate data using automated remote sensing from sites in the mountainous regions of the Western US and site-specific data, maps and graphs provide information on snow water equivalent, snow depth, precipitation, and temperature in hourly, daily, monthly and yearly increments)
  - Water Supply Outlook for major reservoirs
  - Spring and Summer Streamflow Forecasts for major basins
  - Snowpack and snow-cover maps
- Physical Hydrology (Dingman, 2002)
- Dynamic Hydrology (Eagleson, 1970)
- Snow Hydrology (US ACE, 1956)
- Runoff from Snowmelt (US ACE, 1960)

**II. Frozen Soil Modeling**

**Relevance**

Soil frost can drastically reduce the infiltration rate through a soil, break down soil aggregates and decrease soil strength, and heave large buildings and roads. Water that freezes in the soil expands about 9% by volume and decreases infiltration capacity of the soil. Peak runoff and erosion events in the Palouse region occur when it rain or snowmelt or both occur on frozen or partially frozen soils. Erosion is greatest when a thin saturated layer of thawed soil exists over an impermeable frozen soil layer.

**Interesting Findings about Frozen Soil**

- Bullard (1954): for every 10 cm of snow, frost depth was reduced by 80%
- Crawford (1957): the frost depth was reduced about 1 ft for each ft of snow
- Steppuhn (1981): a “rule of thumb” in Europe is that for every cm of snow cover, soil temperature in the root zone increases by at least 0.1 °C
- Chandler (1946): forest soils, even when frozen, are more permeable than those in the open field

**Major Types of Frost**

- **Concrete frost**: has many ice lenses that dramatically reduce infiltration, typically found in moist fine-textured soils
- **Porous concrete frost**: concrete frost that has melted internally as well as above and below
- **Granular frost**: infiltration increases when soil is frozen, typically occurring in forest- or dry soils
- **Honeycomb or stalactite**: usually found in forest soils near surface and have no effect on infiltration

**Redistribution of Water to Frozen Front**

As a soil freezes, liquid water from beneath moves into the frozen soil layer due to a concomitant
pressure gradient induced by the temperature gradient. To understand this process we must understand the concept of freezing-point depression and the similitude assumption.

Water can remain in a liquid form in the soil at temperatures well below 0 °C, which is called freezing point depression. The unique characteristics of water’s phase diagram (with a liquid and solid line sloping to the left as a result of water being denser when in liquid phase than in solid phase) implies that higher air pressure favors the liquid state of water. For instance, when the air pressure increases, the freezing point decreases. On the other hand, when the soil water pressure is below the atmospheric pressure (of 1 atm), the freezing point decreases as well. For instance, under a soil water pressure of −12.2 bar, the temperature at which ice crystals begin to form is −1 °C—appreciably below the temperature of 0 °C at which water would freeze if the air and liquid water pressure is 1 atm or 1 bar. A similar phenomenon explains why champagne in a bottle does not freeze when the temperature drops below 0 °C. Once the cork is popped and the pressure is released ice would immediately form in the bottle.

In the champagne bottle the air pressure increases (above the normal atmospheric pressure) while the liquid pressure remains constant, whereas in the soil the air pressure remains constant (at atmospheric pressure) and the liquid water pressure decreases. The latter likens the former in the sense that the liquid water pressure is “relatively low” compared to the air pressure. According to the Clapeyron equation, the freezing point $T_{fp}$ is related to the liquid water pressure $P_l$ (in MPa; note that 1 MPa = 10 bar)

$$T_{fp} = 0.819P_l$$  \hspace{1cm} (18)

Therefore, at a liquid water pressure of −12.2 bar, the freezing point would be $0.819 \times (-1.22) = -1$ °C.

Following a similitude concept, the pressure forces keeping water from freezing in a freezing soil are the same as the pressure forces keeping water from evaporating in a drying soil. Therefore, the relationship between soil water pressure and water content in a non-frozen soil is exactly the same as the liquid water pressure and liquid water content in a frozen soil.

As liquid water moves to the frozen soil layer due to the pressure gradient induced by the natural temperature gradient, ice lenses tend to form. As these ice lenses form, the pressure builds, pressing the surrounding, and the soil begins to heave, which is called frost heaving. Frost heaving can tremendously damage highways and roads. Frost heaving tends to occur in fine textured soils that have a good supply of ground water, e.g., under shallow water-table conditions. Soils in the Palouse region can heave but generally the soil water content is too low and there is not a good supply of ground water.

### Frozen Soil Modeling

Like snowmelt modeling, there are two types of frozen soil modeling.

- Energy-based methods that couple heat and water transport
  - Simultaneous Heat and Water (SHAW) model (Flerchinger, 1989), a detailed 1-d model
  - SOIL model (Lundin, 1990)
  - Benoit and Mostaghimi (1985) who do not consider soil-water transport
- Degree-day methods that do not include water redistribution
  - Stefan equation (Jumikis, 1966)
  - Continuous Frozen Ground Index (Molnau and Bissel, 1983)
BSYSE 456/556 Lecture 4

– Cary et al. (1978)

III. Questions

1. What is the relevance and significance of studying snow hydrology and frozen soils?

2. What are the terms (and their definitions) frequently used in snow hydrologic research?

3. What is a typical value of the density of the new fallen snow?

4. What are the four snowpack metamorphisms?

5. Where could you look for state-wide, site-specific information on snow?

6. What are the major types of frost?

7. Describe the process of water redistribution within a frozen soil. What is freezing point depression?

8. What are the greatest challenges in snowmelt modeling at a watershed scale?

9. Do you know any model(s) that have an adequate snowmelt component?

10. What are the two general approaches to frozen soil modeling?

IV. Example Problem

A hydrologist conducted a snow survey and collected the following data on snowpack depth, snow water equivalent, and average temperature of snowpack (Table E.1). Compute the average depth, snow water equivalent, snow density, cold content and the energy needed for the snowpack to complete the melting process.

<table>
<thead>
<tr>
<th>Point</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth, cm</td>
<td>98</td>
<td>108</td>
<td>102</td>
<td>109</td>
<td>105</td>
</tr>
<tr>
<td>Snow water equivalent, cm</td>
<td>27</td>
<td>30</td>
<td>27</td>
<td>30</td>
<td>29</td>
</tr>
<tr>
<td>Temperature of snowpack, °C</td>
<td>-9</td>
<td>-8</td>
<td>-10</td>
<td>-9</td>
<td>-9</td>
</tr>
</tbody>
</table>

Solution:

1. Average snow depth

\[(98 + 108 + 102 + 109 + 105)/5 = 104 \text{ (cm)}\]

2. Average snow-water-equivalent depth

\[(27 + 30 + 27 + 30 + 29)/5 = 29 \text{ (cm)}\]
3. Average snow density

From Eq 1 (this note), \( \dot{h}_m = \rho_s / \rho_w \cdot \dot{h}_s \), we have \( \rho_s = \dot{h}_m / \dot{h}_s \cdot \rho_w \). The computed density for each point is shown in Table E.2.