Lecture Overview

1. Penman-Monteith equation to determine reference ET: a “big leaf” model
2. Calculations involved in the Penman-Monteith equation
3. Data needs for the Penman-Monteith equation and its simplifications


Calculating the latent heat flux or evapotranspiration (ET) from a full-canopy cropped surface requires integration of transpiration from leaf to canopy while assuming that fluxes from the soil surface are negligible. A simplification consists of treating the entire canopy as a single “big leaf”. A representation of the system with emphasis on bulk surface resistance (canopy resistance) and aerodynamic resistance is given in Fig. 1.

![Fig. 1. Simplified representation of canopy and aerodynamic resistances and flows in a cropped surface (courtesy of Allen et al., 1998).](image)

Combining energy balance and mass transfer equations for this system results in the combination method for ET calculations. We will now derive the Penman-Monteith equation, a biophysically complete formulation with inclusion of canopy control on the transpiration process.

The energy balance can be written as follows

$$R_n - \lambda E - H - G = 0$$  

(1)

where $R_n$, $\lambda E$, $H$, and $G$, all in MJ m$^{-2}$ d$^{-1}$, are the net radiation (MJ m$^{-2}$ d$^{-1}$), latent heat flux, sensible heat flux, and soil heat flux ($\approx$0), respectively.

The steady-state exchange of sensible heat between a crop canopy and surrounding air may be expressed
as

\[ H = \frac{\rho C_p(T_c - T_a)}{r_a} \]  

(2)

where \( \rho \) is atmospheric density (kg m\(^{-3}\)), \( C_p \) is the specific heat capacity of moist air (0.001013 MJ kg\(^{-1}\) °C\(^{-1}\)), \( T_c \) is canopy temperature (°C), \( T_a \) is air temperature (°C), and \( r_a \) is aerodynamic resistance (d m\(^{-1}\)).

On the other hand, the steady-state exchange of vapor (latent heat) between a crop canopy and surrounding air may be expressed as

\[ \lambda E = \frac{\rho C_p[\varepsilon^0(T_c) - e_o]}{\gamma(r_a + r_c)} \]  

(3)

where \( \varepsilon^0(T_c) \) is the saturation vapor pressure at temperature \( T_c \) (°C), \( e_o \) is the actual vapor pressure (kPa), \( r_c \) is canopy resistance (d m\(^{-1}\); a value of 0.00081 d m\(^{-1}\) is used for reference grass), and \( \gamma \) is the psychrometric constant (kPa °C\(^{-1}\)).

The net radiation (\( R_n \)) term can be measured or estimated from solar radiation and temperature (more on this later) assuming that canopy and air temperature are the same. The soil heat flux term (\( G \)) can be estimated from temperature or as a fraction of \( R_n \). For many applications (e.g., daily or larger time-step calculations), \( G \) can be assumed as zero.

One problem that arises in using a combination of Eq 1, 2, and 3 is that \( R_n, H \) and \( \lambda E \) depend on canopy temperature while the latter, in turn, depends on the canopy energy balance. This problem can be solved by iteration, but this is not very practical.

Penman worked out a solution to this problem. Let us consider the saturation vapor pressure function of temperature (see Fig. 2). The following approximation can be written

\[ \varepsilon^0(T_c) - e_o = [\varepsilon^0(T_o) - e_o] + \Delta(T_c - T_o) \]  

(4)

where \( \Delta \) is the slope of the saturation pressure function of temperature (kPa °C\(^{-1}\)). A vapor pressure deficit of the atmosphere (\( VPD, \) kPa) term can be defined as follows

\[ VPD = \varepsilon^0(T_o) - e_o \]  

(5)

Combining Eq 3, 4, and 5, the following expression is obtained for latent heat

\[ \lambda E = \frac{\rho C_p[VPD + \Delta(T_c - T_o)]}{\gamma(r_a + r_c)} \]  

(6)

Since \( T_c \) is unknown, it can be eliminated by combining Eq 6 and 2 to obtain the Penman transform.
Fig. 2. Saturation vapor pressure function of temperature (curve) and definitions for:

To eliminate $H$, we combine Eq 7 and 1 to obtain:

$$\lambda E = \frac{\rho C_p VPD + \frac{\Delta H r_a}{\rho C_p}}{\gamma (r_a + r_d)}$$

(7)

Expanding Eq 8 and dividing the numerator and denominator on the right-hand side by $r_a$ yields:

$$\lambda E = \frac{\rho C_p VPD r_a + \Delta (R_n - G) - \Delta \lambda E}{\gamma (1 + r_c/r_d)}$$

(8)

The next step consists of collecting the latent heat term in the right-hand side of the equation.

$$\lambda E \left[ 1 + \frac{\Delta}{\gamma (1 + r_c/r_d)} \right] = \frac{\Delta (R_n - G) + \rho C_p VPD r_a}{\gamma (1 + r_c/r_d)}$$

(9)

Now, by multiplying both sides by $\gamma (1 + r_c/r_d)$ and solving for $\lambda E$, the Penman-Monteith (P-M) form of
the combination equation (combination of energy balance and energy flux equations) is obtained

$$\lambda E = \frac{\Delta (R_n - G) + \rho C_p VPD/\rho_a}{\Delta + \gamma (1 + r_c/r_a)} \quad (11)$$

where $\lambda E$ is the reference crop ET in MJ m$^{-2}$ d$^{-1}$.

Divide $\lambda E$ by the latent heat of vaporization $\lambda$ to obtain kg m$^{-2}$ d$^{-1}$, which can be readily converted to mm d$^{-1}$. The P-M equation was originally intended for instantaneous or very short time steps. However, practical applications call for daily calculations with little loss of accuracy.

2. Calculations Involved in the Penman-Monteith Equation

The set of equations presented in this section are intended to calculate reference crop ET (height = 0.12 m, canopy resistance = 70 s m$^{-1}$ or 0.00081 d m$^{-1}$) for a full-day (24-h) period using the Penman-Monteith formulation. These equations closely follow the implementation introduced in the *FAO Irrigation and Drainage Paper No. 56* (Allen et al., 1998) on crop ET. The Penman-Monteith equation is written as in Eq 11 above.

Net radiation

$$R_n = R_{ns} - R_{nl} \quad (12)$$

where $R_{ns}$ is the shortwave net radiation and $R_{nl}$ is the isothermal longwave net radiation, both in MJ m$^{-2}$ d$^{-1}$.

$$R_{ns} = (1 - \alpha)R_s \quad (13)$$

where $\alpha$ is albedo ($\alpha=0.23$) and $R_s$ is solar radiation in MJ m$^{-2}$ d$^{-1}$.

$$R_{nl} = f_c f_h \sigma\left(\frac{T_{Kmax}^4 + T_{Kmin}^4}{2}\right) \quad (14)$$

where $f_c$ is the cloudiness factor (unitless), $f_h$ is the air humidity correction factor (unitless), $\sigma$ is the Stefan-Boltzman constant ($\sigma = 4.903 \times 10^{-9}$ MJ m$^{-2}$ d$^{-1}$), $T_{Kmax}$ is the maximum temperature (K), $T_{Kmin}$ is the minimum temperature (K), and

$$f_c = 1.35 \left(\frac{R_s}{R_{so}}\right) - 0.35 \quad (15)$$

where $R_{so}$ is the clear-sky solar radiation (0.75 × extraterrestrial solar radiation), and

$$f_h = (0.34 - 0.14 \sqrt{e_a}) \quad (16)$$

where $e_a$ is the actual vapor pressure (kPa) as previously defined.

The extraterrestrial solar radiation $R_{so}$, in MJ m$^{-2}$ d$^{-1}$, is calculated as follows
where \( K_{sc} \) is the solar constant (118.08 MJ m\(^{-2}\) d\(^{-1}\)), \( d_r \) is the inverse relative distance between the earth and the sun, \( \omega_s \) is the sunset hour angle (rad), \( \theta \) is latitude (rad) = latitude (degrees) \( \times \pi/180 \), \( \delta \) is the solar declination angle (rad), and

\[
d_r = 1 + 0.033 \cos \left( \frac{2 \pi}{365} J \right)
\]

where \( J \) is the day of the year (Julian day), \( \delta = 0.409 \sin \left( \frac{2 \pi}{365} J - 1.39 \right) \), and \( \omega_s = \arccos \left[ -\tan(\theta) \tan(\delta) \right] \).

**Vapor pressure deficit**

\[
VPD = e_s - e_a
\]

where \( e_s \) is the mean saturation vapor pressure (kPa), and \( e_a \) is the actual vapor pressure (kPa).

\[
e_s = \frac{e^0(T_{max}) + e^0(T_{min})}{2}
\]

where \( e^0(T) \) is the saturation vapor pressure at temperature \( T \) (°C) as previously defined.

\[
e^0(T) = 0.611 \exp \left( \frac{17.27 T}{T + 237.3} \right)
\]

The actual vapor pressure can be determined from the dew-point temperature

\[
e_a = e^0(T_{dew}) = 0.611 \exp \left( \frac{17.27 T_{dew}}{T_{dew} + 237.3} \right)
\]

where \( T_{dew} \) is the dew-point temperature (°C) for the parcel of air of interest.

Alternatively, the actual vapor pressure can be determined from the relative humidity (in percentage) as follows

\[
e_a = \frac{e^0(T_{min}) \frac{RH_{max}}{100} + e^0(T_{max}) \frac{RH_{min}}{100}}{2}
\]

For situations where air humidity is not available for the period of interest, \( VPD \) can be estimated from air temperature

\[
VPD = C \cdot VPD_{max}
\]

where \( VPD_{max} \) is the maximum vapor pressure deficit of the day (kPa), and \( C \) is the ratio of mean daily \( VPD \) to maximum \( VPD \) (typical values 0.45–0.50), and
where $a$ is the aridity factor. For the aridity factor, a value of zero can be used for a humid environment and a value of 0.01 can be used for a very arid environment. Both $a$ and $C$ should be calibrated using a minimum of one year of air humidity and temperature data.

**Aerodynamic resistance**

The calculation of aerodynamic resistance assumes a reference crop height ($h_r$) of 0.12 m.

$$\frac{\ln \left( \frac{z_m - d}{z_{om}} \right)}{k^2 U_z} \ln \left( \frac{z_h - d}{z_{oh}} \right)$$

where $r_a$ is the aerodynamic resistance in d m$^{-1}$; $d$ is the zero-plane displacement of wind profile in m; $d = (2/3) h_r = 0.08$ m; $z_m$ is the height of wind speed measurements in m; $z_h$ is the height of temperature and humidity measurements in m; $z_{om}$ is the roughness parameter for momentum in m, $z_{om} = 0.123 h_r = 0.01476$ m; $z_{oh}$ is the roughness parameter for heat and water vapor in m, $z_{oh} = 0.1 z_{om} = 0.001476$ m; $k$ is the Von Karman constant and $k = 0.41$; and, $U_z$ is the wind speed measurement at height $z_m$ in m d$^{-1}$.

For the calculation of ET, wind speed measured at 2 m above the ground surface is required. The following equation can be used to adjust wind speed data obtained from instruments placed at elevations other than the standard height of 2 m.

$$U_z = U_z \frac{4.87}{\ln(67.8 z_m - 5.42)}$$

where $U_z$ is the wind speed at 2 m above the ground surface in m d$^{-1}$.

**Other calculations**

The slope of the saturation vapor pressure function of temperature is given by

$$\Delta = \frac{4098 e^0(T_{mean})}{(T_{mean} + 237.3)^2}$$

The atmospheric density in kg m$^{-3}$ is calculated as

$$\rho = 3.486 \frac{P}{T_{kv}}$$

where $P$ is the atmospheric pressure in kPa, and $T_{kv}$ is the virtual temperature in K, $T_{kv} = 1.01 (T + 273)$.

The psychrometric constant is given by

$$\gamma = \frac{C_p P}{0.622 \lambda}$$
where \( C_p \) is the specific heat capacity of moist air, and \( C_p = 0.001013 \text{ MJ kg}^{-1} \text{ C}^{-1} \), as given earlier. \( \lambda \) is the latent heat of vaporization as previously defined.

The atmospheric pressure in kPa can be estimated as follows

\[
P = 101.3 \left( \frac{293 - 0.0065z}{293} \right)^{5.26}
\]

(31)

where \( z \) is the elevation above the sea level in m.

The latent heat of vaporization \( \lambda \) in MJ kg\(^{-1} \) is given by

\[
\lambda = 2.501 - 0.002361 T_{\text{mean}}
\]

(32)

3. Data Needs for the Penman-Monteith Equation and Its Simplifications

The implementation of the P-M equation requires daily weather data, including maximum temperature, minimum temperature, solar radiation, maximum and minimum relative humidity or dew-point temperature, and wind speed. P-M equation has been incorporated into a number of hydrology models, such as WEPP and CropSyst. A spreadsheet implementation is also accessible online (Allen, 2006).

For conditions where not all required weather variables are available, simplifications are necessary. If we assume that \( \lambda E \) depends mainly on radiation supply, the advection term can then be neglected and a correlation constant (\( \alpha \), ranging 1.26–1.70) is used as a multiplier of the radiation term. In addition, the canopy resistance may be assumed zero. These simplifications yield the Priestley-Taylor equation.

\[
\lambda E = \alpha \frac{\Delta (R_n - G)}{\Delta + \gamma}
\]

(33)

If we further assume that \( G = 0 \) when \( \lambda E \) is averaged over a few days, and that \( R_n \) is proportional to solar radiation \( R_s \) and air temperature \( T_a \), a general expression that is used by several radiation methods to estimate ET is obtained

\[
\lambda E = a(T_a + b)R_s
\]

(34)

Finally, estimations of \( \lambda E \) using correlation with only temperature have also been developed, which include the Blaney-Criddle and the Hargreaves equations. Simplifications relax the input requirement, increasing applicability but decreasing accuracy and often requiring local calibration. Also, while the Penman-Monteith equation can be applied for short-term calculations (hourly or for even shorter durations), the Priestley-Taylor and other radiation methods are used to predict daily transpiration, and, temperature-based methods are used mainly for weekly or monthly calculations.

Study Questions

1. Why is it important to study evapotranspiration? Provide a case application.
2. Try to derive the Penman-Monteith equation by yourself. It is not as hard as you would think. The benefit of this effort on your own is for you to better understand how both energy balance and energy transfer were considered in the Penman-Monteith equation.
3. Why is the Penman-Monteith equation also called the “big-leaf” model? Is this equation for actual or
potential ET?

4. What is the concept of saturation vapor pressure? How to determine it? Sketch the general shape of the function.

5. What is the concept of vapor pressure deficit (VPD)? How to determine VPD?

6. What is the concept of aerodynamic resistance? How to estimate it?

7. What are the data needs for applying the Penman-Monteith equation? In what ways can you simply this model? What are the limitations of the simplified models?

References